

MARVIN SHINBROT: 1928 - 1987

by
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It came as a sudden, unexpected shock to Marvin Shinbrot's many friends and co-workers around the globe when the news of his death spread after September 18th, 1987. Few had known that he was ill, and hardly anybody suspected the seriousness of his lung problem. He had clearly decided not to burden his friends with his condition and to continue to live a normal life for as long as possible - and he did so until the very last day. The summer of 87 saw him teach an extra heavy load; he kept working on many scientific projects, and his contagious enthusiasm for mathematics was unabated. Friends and colleagues received scientific mail from him days after his death.

His absence means a big loss not only for applied mathematics and mathematical physics; it also deprives the mathematical community of a most unusual personality, of a man with strong beliefs who left his mark in mathematics as well as in the advancement of all kinds of ethical and social issues.

Marvin Shinbrot was born on May 30th, 1928, son of Meyer Shinbrot and Sylvia Goldberg Shinbrot, in Brooklyn, New York. He went to high school there until, aged thirteen, he met some students from the Fork Union Military Academy during the annual family vacation in Florida. Their stories enticed him to join this academy, and he rose to the rank of lieutenant upon graduation.

His mathematical talent began to show during these early years, and his year book was signed by all his friends "To the mathematical genius of New York." His attendance at a military school is in stark contrast to his anti-militaristic activities later in his life.

After high school graduation he went to Syracuse University, where he received his undergraduate university education. Among the subjects he enrolled in were courses in German, which he spoke well and which we sometimes used as a "secret language" (when needed) in North America. It was also in Syracuse where he met his first wife.

In 1949 he went on to Stanford for graduate study. For whatever reasons, the graduate curriculum there at that time - or his thesis topic - did not sufficiently inspire him, and he left Stanford in the same year to work for the National Advisory Committee for Aeronautics (NACA) at Moffett Field, in California. He began his work on linear systems theory and integral equations during the years 1949-1957, while working for NACA. From 1957 to 1961 he worked as a research scientist with top secret status for the Lockheed Aircraft Corporation. The publications from these years contain numerous explicit military applications, such as the optimal design and control of gun platforms, optimal guidance of missiles, etc.

Eventually he grew disillusioned with such applications of his work; his social and political conscience stirred. He resumed graduate work at Stanford, this time with a topic of his own choice (his mathematical work will be described in more detail below), and obtained his Ph.D., in record time, in 1960 with a thesis entitled "Difference Kernels." It was the beginning of many years of work on integral equations.

He entered academia. His first appointment was at the University of Chicago, where he worked as an instructor in 1961 and 1962. In 1962 he was appointed Assistant Professor at Berkeley, and the family moved back to California.

These were years full of excitement and disturbance at Berkeley, and Marvin and his wife became actively engaged in the movements for civil rights and against the Vietnam war. At the same time, it was a period of tireless and fruitful research; Marvin's first fundamental contributions to Wiener-Hopf factorization, to the theory of water waves and the Navier-Stokes equations all date back to this period. He began several of his many collaborations, among others the one with S. Kaniel.

In 1965 the Shinbrots moved to Evanston; Marvin became Associate Professor of Mathematics and Engineering Sciences at Northwestern University, and was promoted to full professor in 1967. His scientific output grew steadily and became more diversified. A. Friedman became one of his many collaborators at Northwestern.

The political and social commitment of the Shinbrots continued. Marvin saw injustice and corruption in his surroundings, even inside the University, during these years of social and racial unrest. Unlike most people he disregarded his own interests and spoke out for his convictions, putting his position and even his family life in jeopardy. He acted as faculty advisor to "Students for a Democratic Society;" he personally handed out flyers protesting the Vietnam war and civil rights violations; he spoke out against the role of the university as a slumlord. This attitude brought him many enemies - he was branded "the most dangerous man in Northwestern University, possibly the United States," by a reactionary professor; he was gassed at the police riots associated with the Democratic National Convention of 1968; his family was harassed - they received bomb threats and rocks through their windows. Marvin's mind was unbent. He had decided what was right and what was wrong, and he would stand up for his opinion, personal consequences be damned.

Clearly such a man had to evoke emotions, and he was thoroughly respected - if sometimes disliked - by faculty and students alike. Eventually, the president of Northwestern University discontinued his contract with the argument that Marvin had taken an unauthorized leave of absence.

This was, however, only the last, formal reason why Marvin left Northwestern for good in 1972. Two years earlier, the escalation in political violence in the U.S. and the fact that his two sons were approaching draft age had already brought the family to Canada. Marvin's first appointment there was as Visiting Professor at the University of British Columbia, and there followed other temporary appointments at Toronto and Oxford. Finally, after his contract at Northwestern University had been discontinued, he came to the University of Victoria. His first marriage had also broken up during this complicated period of his life.

As he settled down in Victoria, his private life calmed down, but his professional life continued to be dominated by countless research projects. It is remarkable that his research record showed no slowdown during the last, rather difficult years at Evanston. Now his mathematical interests shifted once more, this time towards the kinetic theory of gases. Once he explained to me how this happened. He believed that the deep questions that remained unsolved in fluid dynamics (such as uniqueness for the Navier-Stokes equations) could only be solved by insights from a more fundamental level, and consequently began to study the Boltzmann equation.

He traveled extensively. The need to reconcile his established position in life with the university career of his second wife, his love for cosmopolitan life and his general restlessness brought him to move back and forth between Victoria and Montreal (where he held for several years an appointment at the Centre des Recherches Mathématiques Pures et Appliquées) and, later on, between Victoria and Santa Barbara. Every year he visited Europe, where he had strong mathematical contacts in Germany and Italy. He maintained contacts with the Mexican mathematical community, and his last trip before his death was a visit to Mexico City.

An assessment of Marvin Shinbrot's mathematical legacy is a difficult task because of the diversity and size of his contribution. He wrote more than 60 research papers (some of which are yet to appear) and a graduate textbook on fluid dynamics.

His mathematical work can be conveniently classified into eight different categories; 1) contributions on (linear) systems theory (1956-1960), 2) work on Wiener-Hopf factorization methods (1959-1973), 3) contributions to functional analysis, in particular spectral decomposition and fixed point theorems (1963-1974), 4) analytical studies of the Navier-Stokes equations (1966-1976), 5) mathematical analysis of water waves (1962-1987), 6) studies on capillarity and drops (1983-1987), 7) kinetic theory of gases (1976-1987) and 8) other, in particular popular publications.

A detailed description of all the research done in so many articles is beyond the scope of this biography. I choose to describe Marvin's work on topics 2,5 and 7, which were of particular interest to him, in detail, and only give short remarks concerning the other work. This selection is completely subjective and is in no way an assessment of the significance of the chosen or deleted papers. Another biographer would possibly have chosen differently.

The first mathematical publications of Marvin Shinbrot concerned the optimization of time-varying linear systems. In [S2], for example, he investigates which impulse response function $g(\tau, t)$ will minimize

$$\text{average } \langle [\mu(t, P) - x(t; P, Q)]^2 \rangle, \quad \text{where}$$

$$x(t; P, Q) = \int_0^t g(t, \tau) \cdot i(\tau; P, Q) d\tau,$$

and the input $i(\tau; P, Q)$ is a sum of a message function $m(t; P)$ and a noise function $n(t; Q)$. Questions of this type are relevant in numerous engineering applications, and they have military applications: One of the examples quoted in [S2] is the optimal design of a missile guidance system.

Several publications dealt with linear systems, until, at about the time of his return to Stanford, Shinbrot's interest shifted towards integral equations. In retrospect, it is strange that he left the field of linear systems so shortly before the significant advances made by Kalman and Bucy [1], and so shortly before the computer revolution of the sixties. His disillusionment with military applications may be a part of the explanation.

I believe that his entrance into the field of Wiener-Hopf factorization techniques is marked by a paper "A Class of Difference kernels" [S7], in which he gave conditions on the Fourier transform of a function k such that the integral equation of the first kind

$$\phi(x) = \int_{-1}^1 k(|x-y|) f(y) dy, \quad -1 < x < 1$$

has a unique solution for every $\phi \in C^2[-1, 1]$. This paper was the first in a series in which criteria for the solvability of equations

$$PJPF = Ph$$

were investigated. In a more general setting, P is a projection and J is a linear operator of special type in a Hilbert space, e.g. $J = H_- + NH_+$ (N is a self-adjoint operator of single sign, and H_- , H_+ are projections with $H_+H_- = H_-H_+ = 0$, and $H_- + H_+ = id$). The most general result was reached jointly with A. Devinatz in [S24]: Let H be a Hilbert space, L a closed subspace of H and $P : H \rightarrow L$ the orthogonal projection. Moreover, let A be an invertible linear operator on H , and define the general Wiener-Hopf (Toeplitz) operator $T_P(A)$ by $PA|_L$. Under these conditions, $T_P(A)$ is invertible if and only if there is a factorization $A = A_-A_+$, where A_- , $A_+ \in B(H)$ are invertible operators and the restrictions $A_- : L^\perp \rightarrow L^\perp$, $A_+ : L \rightarrow L$ are also bijective. The inverse $[T_P(A)]^{-1}$ is then given by $A_+^{-1}PA_-^{-1}|_L$.

This result exemplifies the flavor of Shinbrot's work on Wiener-Hopf factorization; later investigations concentrated on generalizations to unbounded operators.

The significance of this work is demonstrated by the fact that Wiener-Hopf factorization, as described in [S24], has become a powerful and widely used tool in operator theory, the theory of partial differential equations and mathematical physics. Shinbrot's strong influence on this development is documented in the book "General Wiener-Hopf Factorization Methods" by F.O. Speck [2].

In no other field did Marvin Shinbrot work longer than on the theory of water waves. He began to study waves in 1961 and continued to contribute to the field until his death. One can certainly say that water waves were one of his passions.

His first result in the field was a derivation of the shallow water equations from the linear equations for water waves in two dimensions, provided that these linear equations have sufficiently smooth solutions (see [S8]). The linear water wave problem is this:

In a domain $-\infty < x < \infty$, $-\epsilon \cdot h(x; \epsilon) < y < 0$, find a velocity potential $\varphi(x, y; t)$ such that $\varphi_{xx} + \varphi_{yy} = 0$ and such that the Bernoulli boundary condition $g \cdot \varphi_y + \varphi_{tt} = 0$ at $y = 0$ and the boundary condition $\varphi_y + \epsilon \cdot h_x \cdot \varphi_x = 0$ at $y = -\epsilon h$ are satisfied. Here $u = \varphi_x$ and $v = \varphi_y$ are the horizontal and vertical components of the fluid velocity respectively, and $\eta(x; t) = -\frac{1}{g}\varphi_t(x, 0; t)$ is the free surface of the water. In this first paper, conditions on φ are given such that as $\epsilon \rightarrow 0$, η is approximated by a solution $\hat{\eta}$ of the shallow water equation in the case of a simple harmonic motion:

$$\epsilon (h\hat{\eta}_x)_x + \frac{\sigma^2}{g}\hat{\eta} = 0.$$

As mentioned, the existence of a sufficiently smooth solution to the linearized equations of water waves has been assumed here. This existence question was then treated, in a much more general form, jointly with A. Friedman in [S19] and [S22]. Several methods (based on separation of variables) to prove the existence of solutions to this problem were developed. The solutions found in [S19] were not classical because the boundary condition $\varphi_y + \varphi_{tt} = 0$ on the “free” surface Γ could only be shown to be satisfied weakly. This drawback was removed in [S22] for the special case where Γ is the hyperplane $y = 0$.

Subsequent papers, mainly joint with J. Reeder, dealt with the full nonlinear problem of water waves under gravity. The problem of the free surface was cleverly circumvented by passing to Lagrangian coordinates. This reformulation of the problem allowed them to prove local existence theorems, provided that the bottom of the domain occupied by the fluid is uniformly analytic and that the initial data are also uniformly analytic.

Later contributions to the theory of water waves concerned steady, solitary and periodic waves. Marvin departed boldly from the conformal mapping approach first introduced by Levi-Civita and introduced a mapping that transformed (not conformally) the original, unknown domain by brute force into a fixed strip (or layer in three dimensions). This distorted the Laplacian, but made the problem accessible to all kind of analytical tools. The idea was used to study solitary waves with surface tension (in two dimensions [S49]), periodic flows over periodic, nearly flat bottoms [S43], nonlinear wave interaction in water of constant depth [S46] and, finally, the Wilton ripple phenomenon: It was shown that a periodic wave train spontaneously doubles its frequency as the surface tension parameter moves through a certain critical value.

The idea to map the unknown domain occupied by the fluid into a fixed strip is original, simple and typical of Marvin’s way of thinking: He first found out what others had done, but he refused to follow their paths - he had to understand a new problem from the ground up and find his own ways, thereby gaining new insights.

The third topic to be described here in detail, and also one of Shinbrot's passions, is the kinetic theory of gases, which occupied him for fifteen years and was his favorite in the eighties. In the sixties, he (primarily jointly with S. Kaniel) had contributed greatly to the theory of existence and regularity for the Navier-Stokes equations; eventually, the fundamental open questions in this field led him to another level of investigation: He concluded that insight on fluid dynamics might be obtained by studying the relevant phenomena on the more basic kinetic level. Consequently, he began to investigate the theory of the Boltzmann equation.

The first results, partly joint with J. Schnute, gave derivations of the boundary conditions for fluids from kinetic theory [S32]. Soon, Marvin's interest concentrated on the existence question for the full Boltzmann equation. In a joint work with S. Kaniel [S39], they obtained a local existence and uniqueness result for very general interaction kernels and boundary conditions. This was not the first existence theorem for the Boltzmann equation, but they invented a new, original method based on monotone approximations, which has become known as the "Kaniel-Shinbrot iteration scheme." Later studies concentrated on the notoriously difficult global existence question; Marvin became literally obsessed with this problem, and it brought him great satisfaction when we proved, together, global existence and uniqueness at least in the case of a rare gas cloud in all space [S55]. It is a sad coincidence that he died only months before the first weak global existence result for large data (or small mean free paths) was announced by R. DiPerna and P.L. Lions [3]. This result, even though it leaves many questions (e.g., uniqueness) open, was what Marvin had sought for years.

He had a vital interest in the derivation of kinetic equations from basic principles; one of his papers on the Boltzmann equation [S56] gives a simplified proof of Lanford's validity result for the B.E. [4], and he investigated the so-called Boltzmann hierarchy in the summer of 1987.

A description of Shinbrot's work would be incomplete without a comment on what kind of expositor he was. It was a pleasure to read his papers (unlike so many others nowadays); the problem was always carefully introduced from its background - usually a physical background. The meaning of every equation was well explained, and the intuition of the author was almost magically transferred to the reader. This was not done at the expense of rigor - everything was carefully defined, formulated in terms of theorem and proof, and worked out in sufficient detail. To the non-expert who wants to get a glimpse of this art of paper-writing, I recommend Marvin's proof of the Cauchy-Kowalewskaya theorem (joint with R.R. Welland, see [S36]).

This extraordinary skill in exposition was also prevalent in seminar and colloquia talks, and it shows up most prominently in the few popular articles Marvin has written. In [S15], he makes it his task to explain the contraction mapping principle and the Brouwer fixed point theorem to a general audience, and what a job he does! For example, the stirring of a cup of tea is used to illustrate the meaning of the Brouwer fixed point theorem.

In fact, one of Marvin's last articles was written for a general audience and appeared in "The Sciences" [S61]. He took up the old hoary question of the origin of irreversibility, so notorious in kinetic theory, and discussed it in view of modern results. Even though I do

not agree with all the assertions made in this paper (I believe that the significance of my own contribution to the question (see [5] for a complete treatment) is overemphasized), the article is a pleasure to read.

What kind of person was he? He certainly considered himself an applied mathematician, but he had the tools of pure mathematics readily at hand. His interests were really in mathematical physics, and it is in this sense that his research was applied - it had a direct relation to reality. Actually, he considered the distinction between pure and applied mathematics as rather insignificant.

He made many mistakes, and never denied it. He had a target in mind and followed his intuition towards that target, looking for links but never forgetting the target. He readily produced manuscripts that were incomplete or contained faulty reasonings, to be used for discussion. Even these manuscripts usually led to significant progress.

His computational speed was impressive. A student once complained to me that "he always does five steps in one," and I understood, for he did the same to me in our frequent discussions. He was not afraid of complicated integrals and had a photographic memory of methods to tackle them. He loved to work out a perturbation analysis for a complex problem. He produced lengthy manuscripts overnight on his word processor.

He hated to work alone. This explains why he published so many things jointly. He could not bear to be alone in his office - he constantly had the urge to share his ideas and his enthusiasm with others, students, co-workers, visitors. In mathematical discussions, he was extremely outgoing (but never offending). He would always encourage students and collaborators, thereby strengthening their self-confidence.

The number of his collaborators is so large that I cannot possibly list them all (I believe I don't even know all of them by name). Many of them have greatly contributed to this biography, by information which was not previously available to me. I would like to thank in particular John Reeder, Erhard Meister, Frank-Olme Speck, Robert Finn, Chandler Davis, Avner Friedman, Jon Schnute, Carlo Cercignani, Albert Hurd, Jim Donaldson and Shmuel Kaniel. I am indebted to Troy Shinbrot and Mrs. Sylvia Goldberg for supplying me with the biographical information.

A man like Marvin must have an epitaph. I cannot do better than quote from a letter sent to me by his son Troy, in which he wrote:

"Of all things, my father loved three things best. He loved achievement, he loved truth, and he loved wine.

He loved not only to achieve, he loved achievement itself - hence his undying interest in the historymakers of Science.

He loved truth; his actions at Northwestern exemplify this.

And of course he loved wine. I don't simply mean that superficially, although it is certainly true by itself. I mean he loved life and its sensual pleasures. He would work and plan a meal for days without hesitation - because the full enjoyment of life was crucial to him."

All those who knew Marvin Shinbrot know that this describes, in a few lines, the intense person he was. He will be greatly missed.

List of Publications by Marvin Shinbrot

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